ALGEBRAIC TOPOLOGY I WS23/24, HOMEWORK SHEET 11

DEADLINE: JANUARY 19, 2024

Problem 1. Let X be an n-dimensional CW-complex. Show that the map

$$\mathrm{Vect}^m_{\mathbb{R}}(X) \to \mathrm{Vect}^{m+1}_{\mathbb{R}}(X)$$
$$\xi \mapsto \xi \oplus \epsilon$$

is a bijection for $m > n$, and a surjection for $m = n$.

Problem 2. A smooth manifold M is said to admit a *field of tangent k-planes* if its tangent bundle admits a subbundle of dimension k. Show that $\mathbb{R}P^n$ admits a field of tangent 1-planes if and only if n is odd. Show that $\mathbb{R}P^4$ and $\mathbb{R}P^6$ do not admit fields of tangent 2-planes.

(Hint: If $n = 2k - 1$ is odd, the 1-dimensional subbundle can be chosen to be trivial. Use the smooth covering $S^{2k-1} \to \mathbb{R}P^{2k-1}$ and the complex structure on $\mathbb{R}^{2k} \cong \mathbb{C}^k$ to construct this trivial subbundle.)